Time-based deterministic model of information security of a business organization

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This paper discusses information security of a business organization using a time-based mathematical deterministic model. The model addresses key features of a business organization from the point of view of information security and calculates the level of information security based on quantitative values. Next, the introduced model is used to evaluate the level of information security that could be achieved for known threats within a given budget. For this reason, an optimization problem of safeguard implementation is formulated and an optimization method based on dynamic programming is used to address the issue. Two samples, local and global security metrics, defined in the model are described and one of them is used in optimizing safeguard implementation.

Keywords: information security, deterministic model, dynamic programming.

1. Introduction

In a business organization, there are business processes and information systems supporting them [4, 5]. Each business process should lead to a presupposed financial result.

Information systems are vulnerable to a great number of risks [5]. Risks can occur repeatedly and each time the level of threat may be different. To counter threats business organizations use information system safeguards. Each safeguard has distinct characteristics determining the intensity of protection [5].

An organization’s security policy determines when and which safeguard to apply to the information system and at what intensity. Some safeguards are implemented at the beginning of the business process. Later, additional safeguards can be implemented. The length of the protection period is known. The occurrence of a threat causes losses to the business process [5].

Costs of safeguard comprise the cost of implementation and maintenance costs (calculated for each time period). Each organization has a budget. In each time period the budget of a business organization is increased by the anticipated financial effects of business processes and reduced by costs of implementation and maintenance of safeguards as well as losses resulting from the occurrence of threats. A business organization should establish a financial limit for implementing and maintaining safeguards [5].

2. Deterministic mathematical model of information security

In the model, according to the rules of constructing mathematical models [2], we consider a period of time divided into periods labelled with variables $t=\overline{1,T}$. We will use P to indicate the number of business processes. There is a financial effect connected with each business process. We will denote by $L_p(t)$ the financial effect for a business process $p=\overline{1,P}$ in time period t, calculated without safeguards implemented and without danger events. In this particular situation, the financial effect may occur in only one period of time, when the process is completed in this period or earlier, and followed by a one-off payment.

Information systems are frequently exposed to threats [3, 5]. We will use V to indicate the number of types of threats. Threat events have different levels of intensity. We will use $X_v(t)$ to denote the intensity level of threat $v=\overline{1,V}$ in time period $t=\overline{1,T}$. The vector of intensity levels in time period t is designated as $\mathbf{X}(t) \triangleq \left[X_1(t), X_2(t), ..., X_V(t)\right]$.

The process of threat occurrence can be described by

$\mathcal{X}(t) \triangleq \langle \mathbf{X}(t), \mathbf{X}(t-1), \mathbf{X}(t-2), ..., \mathbf{X}(1) \rangle$,

which means a history of threat occurrences until time period t.

Organizations use information system safeguards to counter threats [3, 5]. An organization's security policy determines which safeguards will be implemented, at what intensity and when. There are safeguards implemented before the start of a business process. After the occurrence of a threat, additional safeguards can be implemented. We will use I to denote the number of different types of safeguards. Each safeguard has its distinct intensity. We will designate through $\overline{O}_i$ the intensity of the safeguard $i = \overline{1,I}$, which safeguard will have following successful implementation until the end of its functioning. We will use $O_i(t)$ to indicate the intensity of safeguard i in time period $t = \overline{1,T}$. The vector of safeguard intensity in time period t is marked as $\mathbf{O}(t) \triangleq [O_1(t), O_2(t), ..., O_I(t)]$. We assume that safeguard intensity does not change. After safeguard ceases to function, its intensity equals 0. The process of safeguard intensity can be described by

$\mathcal{O}(t) \triangleq \langle \mathbf{O}(t), \mathbf{O}(t-1), \mathbf{O}(t-2), ..., \mathbf{O}(1) \rangle$,

which means a history of safeguard intensity until time period t. Implementation of a safeguard can start at any moment.

We will use $s_i(t) \in \{0,1\}$ to indicate the decision to implement safeguard i in time period t, where $s_i(t) = 1$ means the opposite. The vector of the decision to implement safeguard measures in time period t is marked as $\mathbf{s}(t) \triangleq [s_1(t), s_2(t), ..., s_I(t)]$. The matrix of decisions to implement safeguard measures in the analyzed time is marked as

$$\mathbb{S} \triangleq \begin{bmatrix} s_1(1) & ... & s_I(1) \\ ... & ... & ... \\ s_1(T) & ... & s_I(T) \end{bmatrix}_{TxI}.$$

Let the time of safeguard implementation i in the case of implementation beginning in time period t, last $G_i(t)$ period of time. The safeguard will start functioning in time period $t + G_i(t)$. During implementation the safeguard is not active. Time of implementation can be different in different time periods, e.g. during the implementation of the information system it can be lower than during the maintenance phase. In each time period, the organization can start the implementation of a safeguard i only once, which means that:

$$\forall t = \overline{1,T}: \sum_{n=t-G_i(n)+1}^{t} s_i(n) \le 1$$

Information about implementation of safeguard i in time period t can be described as follows:

$$\alpha_i(t) = \sum_{n=\overline{1,T}} [s_i(n) = 1 \wedge 0 \le t - n < G_i(n)]^{1}$$

where $\alpha_i(t) = 1$ means, that safeguard i is being implemented in time period t, and $\alpha_i(t) = 0$ means the opposite.

Each safeguard acts for a limited period of time. After this time the safeguard stops functioning. We will denote by $r_i(t)$ the number of periods of safeguard functioning i if the implementation begins in time period t. This number can be different in different time periods. The end of the functioning of safeguard i is expressed as $t + G_i(t) + r_i(t)$. In each time period, safeguard i can function only once, which means that:

$$\forall t = \overline{1,T}: \sum_{n=t-G_i(n)-r_i(n)+1}^{t-G_i(n)} s_i(n) \le 1$$

Information about the functioning of safeguard i in time period τ can be expressed as follows:

$$\beta_i(t) = \sum_{n=\overline{1,T}} [s_i(n) = 1 \wedge$$
$$\wedge\, 0 \le t - n - G_i(n) < r_i(n)]$$

where $\beta_i(t) = 1$ means that the safeguard i is active in time period t, and $\beta_i(t) = 0$ means the opposite.

The intensity of safeguard i in time period t can be described as:

$$O_i(t) = \overline{O}_i \cdot \sum_{u=1}^{t} s_i(u) \cdot \sum_{a=0}^{r_i(u)-1} [t = u + G_i(u) + a]$$

Implementation or functioning of some safeguard measures can exclude the implementation or functioning of others, e.g. two firewalls cannot be installed on a single machine. Our model takes into account three different types of exclusions of simultaneity:

1. Functioning of one safeguard and functioning of another safeguard.
2. Implementation of one safeguard and functioning of another safeguard.

[1] $[\varphi] \in \{0,1\}$ is the Iverson bracket, where φ is a statement that can be true or false.

3. Implementation of one safeguard and implementation of another safeguard.

We will use ${}^{I}\psi_i^j \in \{0,1\}$ to denote an exclusion of simultaneous functioning of safeguard i and safeguard j, where ${}^{I}\psi_i^j = 1$ means that safeguard i cannot simultaneously function with safeguard j, and ${}^{I}\psi_i^j = 0$ means that safeguard i can function simultaneously with safeguard j.

We will designate through ${}^{II}\psi_i^j \in \{0,1\}$ an exclusion of simultaneous implementation of safeguard i and functioning of safeguard j, where ${}^{II}\psi_i^j = 1$ means that safeguard i cannot be implemented when safeguard j is active, and ${}^{II}\psi_i^j = 0$ means that safeguard i can be implemented when safeguard j is active.

We will use ${}^{III}\psi_i^j \in \{0,1\}$ to denote an exclusion of simultaneous implementation of safeguard i and safeguard j, where ${}^{III}\psi_i^j = 1$ means that safeguard i cannot be implemented when safeguard j is being implemented and ${}^{III}\psi_i^j = 0$ means that safeguard i can be implemented when safeguard j is being implemented. The exclusion of simultaneous functioning of safeguards can be described as follows:

$$ {}^{I}\Psi \triangleq \begin{bmatrix} {}^{I}\psi_1^1 & \dots & {}^{I}\psi_1^I \\ \dots & \dots & \dots \\ {}^{I}\psi_I^1 & \dots & {}^{I}\psi_I^I \end{bmatrix}_{IxI} . $$

The exclusion of simultaneous implementation and functioning of safeguards can be described as follows:

$$ {}^{II}\Psi \triangleq \begin{bmatrix} {}^{II}\psi_1^1 & \dots & {}^{II}\psi_1^I \\ \dots & \dots & \dots \\ {}^{II}\psi_I^1 & \dots & {}^{II}\psi_I^I \end{bmatrix}_{IxI} . $$

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If there is no possibility of simultaneous functioning of safeguards *i* and *j*, the following condition must be true:

$$ \forall t = \overline{1,T} : \beta_i(t) + \beta_j(t) \le 2 - {}^{I}\psi_j^i \qquad (1) $$

If there is no possibility of simultaneous functioning of safeguards i and implementation of safeguard j, the following condition must be true:

$$ \forall t = \overline{1,T} : \alpha_i(t) + \beta_j(t) \le 2 - {}^{II}\psi_j^i \qquad (2) $$

If there is no possibility of simultaneous implementation of safeguards i and j, the following condition must be true:

$$ \forall t = \overline{1,T} : \alpha_i(t) + \alpha_j(t) \le 2 - {}^{III}\psi_j^i \qquad (3) $$

Cost of safeguard comprises the costs of implementation and maintenance costs. Depending on when an organization starts implementation, these costs fall into different categories. Our model assumes that these costs are grouped into one *cost* variable. We will use $e_i^p(n,t)$ to denote the cost of safeguard i for a business process p in time period t, if implementation started in time period n. For a business organization, the financial aspect of information security is the most important factor [3, 5]. We will denote by $K_p(t,\mathbb{S})$ the cost of safeguard implemented for a business process p till time period t using a matrix of implementation decisions described by $\mathbb{S}$ (cumulative cost). We will use $K_p(0,\mathbb{S}) = 0$ to denote no impact on safeguards implemented before the analyzed time is considered. Hence, the cumulative cost of safeguards for a business process p can be described as follows:

$$ K_p(t,\mathbb{S}) = \sum_{i=1}^{I} \overline{O}_i \cdot \sum_{a=1}^{t} s_i(a) \cdot \sum_{u=a}^{t} e_i^p(a,u) $$

Each occurrence of a threat has financial impact on business processes [5]. We will use $\Phi_p(\mathcal{X}(t),\mathbb{S})$ to denote the value that reduces the financial effect of a business process p in time period t, with a history of threat occurrences $\mathcal{X}(t)$, and with an implementation decision matrix $\mathbb{S}$. We will denote by $M_p(t,\mathbb{S})$ the change of the financial effect for a business process p until time period t, with an implementation decision matrix $\mathbb{S}$ (cumulative). It will be assumed that $M_p(0,\mathbb{S}) = 0$. The change of a financial effect takes into account the cost of threat occurrences $\Phi_p(\mathcal{X}(t),\mathbb{S})$, and for a business process p in time period t it can be described as:

$$ M_p(t,\mathbb{S}) = M_p(t-1,\mathbb{S}) + \Phi_p(\mathcal{X}(t),\mathbb{S}) $$

Overall, $\Phi_p(\mathcal{X}(t),\mathbb{S})$ depends on:

1. History of threat occurrences. Occurrence of a threat in the past can influence financial loss in the current time period.
2. History of implemented safeguards.

In most cases threat history can be ignored. Below are examples of the function $\Phi_p\left(\mathcal{X}(t),\mathbb{S}\right)$:

1. As constant costs (paid regardless of threat level)

$$\Phi_1\left(\mathcal{X}(t),\mathbb{S}\right)=B\cdot\operatorname{sgn}\left(\mathcal{X}_1(t)\right)\cdot\left(1-\operatorname{sgn}\left(\overline{O}_1\cdot\sum_{u=1}^{t}s_1(u)\cdot\left[u+G_1(u)\le t<u+G_1(u)+r_1(u)\wedge t<\min\left\{T,u+G_1(u)+r_1(u)\right\}\right)\right)$$

2. As linear dependence on threat level

$$\Phi_1(\mathcal{X}(t),\mathbb{S})=\operatorname{sgn}\left(\mathcal{X}_1(t)\right)\cdot\left(B\cdot\mathcal{X}_1(t)-B\cdot\left(\overline{O}_1\cdot\sum_{u=1}^{t}s_1(u)\cdot\left[u+G_1(u)\le t<u+G_1(u)+r_1(u)\wedge t<\min\left\{T,u+G_1(u)+r_1(u)\right\}\right]\right)\right)$$

3. As complex dependence on threat level

$$\Phi_1\left(\mathcal{X}(t),\mathbb{S}\right)=\operatorname{sgn}\left(\mathcal{X}_1(t)\right)\cdot\left(B\cdot\mathcal{X}_1(t)-B\cdot\left(\overline{O}_1\cdot\sum_{u=1}^{t}s_1(u)\cdot\left[u+G_1(u)\le t<u+G_1(u)+r_1(u)\wedge t<\min\left\{T,u+G_1(u)+r_1(u)\right\}\right]\right)\right)+B\cdot\operatorname{sgn}\left(\mathcal{X}_1(t)\right)\cdot\left(1-\operatorname{sgn}\left(\overline{O}_1\cdot\sum_{u=1}^{t}s_1(u)\cdot\left[u+G_1(u)\le t<u+G_1(u)+r_1(u)\wedge t<\min\left\{T,u+G_1(u)+r_1(u)\right\}\right]\right)\right)$$

Each organization has a budget [5]. The budget depends on financial effects, safeguard costs and financial losses. We will use $\Gamma(t,\mathbb{S})$ to denote the budget of an organization in time period t, with an implementation matrix $\mathbb{S}$. The budget can change in each time period. It is increased by financial effects of business processes, and reduced by safeguard costs and financial losses. The budget of an organization in time period t can be described as follows:

$$\Gamma(t,\mathbb{S})=\Gamma(t-1,\mathbb{S})+\sum_{p=1}^{P}L_p(t)-\sum_{p=1}^{P}\Phi_p\left(\mathcal{X}(t),\mathbb{S}\right)-\sum_{p=1}^{P}\sum_{i=1}^{I}\overline{O}_i\sum_{n=1}^{t}s_i(n)\cdot e_i^p(n,t)$$

We will use $\overline{\Gamma}(t)$ to denote the safeguard budget (budget for implementation and maintenance of safeguards). $\overline{\Gamma}(t)$ can be set for each time period and initially can be greater than $\Gamma(t,\mathbb{S})$ Below we discuss two examples of $\overline{\Gamma}(t)$. In a business organization, the safeguard budget is known from the beginning and its set for each time period, however, sometimes it can depend on the organization's budget, e.g. $\overline{\Gamma}(t)=\frac{1}{2}\Gamma(t,\mathbb{S})$. In time period t the organization cannot spend on safeguard implementation and maintenance more than the value of the safeguard budget for time period t increased by untapped safeguard budget from previous time periods. Thus, the following condition must be true:

$$\forall t=\overline{1,T}:\sum_{p=1}^{P}\sum_{i=1}^{I}\overline{O}_i\sum_{n=1}^{t}s_i(n)\cdot\sum_{u=n}^{t}e_i^p(n,u)\le\sum_{u=1}^{t}\overline{\Gamma}(u) \tag{4}$$

Our model allows to define local and global metrics. In this paper two local and two global metrics are defined. We will use $b_i(t)$ to denote time periods when implementation of safeguard i started, which is being implemented in time period t, where $b_i(t)=0$ means the opposite. Let us note that $b_i(t)$ can be calculated as $b_i(t)=\sum_{n=t-G_i(n)+1}^{t}n\cdot s_i(n)$. We will use $c_i(t)$ to denote time periods when implementation of the safeguard i (functioning in time period t) started, where $c_i(t)=0$ means the opposite. Let us note that $c_i(t)$ can be calculated as

$$c_i(t)=\sum_{n=t-G_i(n)-r_i(n)+1}^{t-G_i(n)}n\cdot s_i(n).$$

The first local metric is a financial effect in time period t, reduced by cost of safeguard and financial loss in time period t, with implementation matrix *S*. This metric is marked as $\gamma(t,\mathbb{S})$ and can be calculated as follows:

$$\gamma(t,\mathbb{S})=\sum_{p=1}^{P}\left(L_p(t)-\Phi_p(\mathcal{X}(t),\mathbb{S})-\sum_{i=1}^{I}\left[b_i(t)>0\right]\cdot\overline{O}_i\cdot e_i^p\left(\mathrm{b}_i(t),t\right)-\sum_{i=1}^{I}\left[c_i(t)>0\right]\cdot\overline{O}_i\cdot e_i^p\left(c_i(t),t\right)\right) \tag{5}$$

Let us note that $\sum_{t=1}^{T}\gamma(t,\mathbb{S})=\Gamma(T,\mathbb{S})$, where $\Gamma(T,\mathbb{S})$ is also a global metric.

The second local metric is a number of threats causing financial loss in time period t, with implementation matrix *S*. This metric is described as $\lambda\left(t,\mathbb{S}\right)$ and can be calculated as follows:

$$\lambda\left(t,\mathbb{S}\right)=\sum_{v=1}^{V}\operatorname{sgn}\left(\sum_{p=1}^{P}\Phi_p\left(\begin{bmatrix} X_1(1) & \cdots & X_v(1) & \cdots & X_V(1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_1(t-1) & & X_v(t-1) & & X_V(t-1) \\ 0 & \cdots & X_v(t) & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}_{V\times T},\mathbb{S}\right)\right)$$

The global metric connected to $\lambda\left(t,\mathbb{S}\right)$ is the number of threats causing financial loss in analyzed time. This metric is described as $\Lambda\left(\mathrm{T},\mathbb{S}\right)$ and can be calculated as follows:

$$\Lambda\left(\mathrm{T},\mathbb{S}\right)=\sum_{n=1}^{T}\lambda(n,\mathbb{S})$$

These are only examples of metrics which can be used later to perform the optimization of safeguard implementation.

3. Optimization of safeguard implementation

Safeguard implementation is designed to maximize the level of information security of a business organization throughout the analyzed period. This section illustrates how to select security safeguards knowing the history of threats. The result of this analysis can be a starting point to compare the effects of the undertaken safeguards to the effects of optimal safeguard implementation. Threat history can also be used to predict future actions. Knowing the history of threats in the analyzed time period, the optimization problem of global metric $\Gamma\left(T,\mathbb{S}\right)$ can be defined as follows:

Designate:

$$\mathbb{S}^*=\begin{bmatrix} s_1^*(1) & \ldots & s_1^*(T) \\ \ldots & \ldots & \ldots \\ s_I^*(1) & \ldots & s_I^*(T) \end{bmatrix}_{IxT}\in\Omega$$

to maximize:

$$\Gamma\left(T,\mathbb{S}^*\right)=\max_{\mathbb{S}\in\Omega}\sum_{t=1}^{T}\gamma\left(t,\mathbb{S}\right)$$

where $\gamma\left(t,\mathbb{S}\right)$ is given in (5) and Ω – is the set of allowed solutions, which means the set of allowed matrixes $\left[s_i\left(t\right)\right]_{I\times T}\in\left\{0,1\right\}^{I\cdot T}$ with restrictions (1), (2), (3) and (4).

Dynamic programming [1] is one of the methods of solving decision problems, including multistage problems. Dynamic programming involves breaking down a complex problem into a set of simple subproblems. The optimal solution to a problem must be a function of optimal solutions to subproblems. Every subproblem is solved only once. Discrete dynamic programming can be applied when:

- A problem can be divided into periods (stages), in each period an optimal decision must be made.
- A set of applicable states of a process is defined at each stage.
- As a result of decision making, each stage is a transformation of the current state into the next level stage.
- Following Bellman's principal of optimality, at each stage, the optimal decision for next stages is independent of decisions taken in the previous stages.

To apply dynamic programming in a given optimization problem, some assumptions were taken:

- Let the time of deployment be independent of time period of implementation start $\forall i=\overline{1,\mathrm{I}};\forall\,\mathrm{t}=\overline{1,T}:G_i\left(t\right)=G_i$.
- Let the time of functioning be independent of time period number of implementation start $\forall i=\overline{1,\mathrm{I}};\forall\,\mathrm{t}=\overline{1,T}:\mathrm{r}_i\left(t\right)=r_i$.
- Let $\Phi_p(\mathcal{X}\left(n\right),\mathbb{S})$ be independent of past threats.
- Let $\forall i=\overline{1,I}:{}^{II}\psi_i^i=1$, which means that the safeguard cannot be implemented and function in one time period.

We will use $N=T+1$ to indicate the number of stages of dynamic programming in a given optimization problem. The stages are enumerated from 1 to $T+1$. The process state can be defined as $y_n\triangleq\left\langle\mathrm{n},q_1,...,q_I,\mathrm{w}_1,...,\mathrm{w}_I,\mathrm{D}\right\rangle$, where q_i is the number of time periods after which safeguard i stops functioning; w_i is the number of time periods after which safeguard i will start functioning, and D is the financial limit for the cost of implementation and maintenance of safeguards at a given stage $\left(i=\overline{1,I};n=\overline{1,N}\right)$.

We will use

$$\mathbb{Y}_n = \{\langle n, q_1, ..., q_I, w_1, ..., w_I, D\rangle \in$$
$$\in \mathbb{N}^{2I+1} \times \mathbb{R}_0^+ :$$
$$\left(\forall i = \overline{1,I} : \left(\left(q_i \cdot w_i = 0\right) \wedge\right.\right.$$
$$\wedge \left(q_i \le \min\{r_i; T-n\}\right) \wedge$$
$$\left.\left.\wedge \left(\mathrm{w}_i \le G_i\right)\right)\right) \wedge \left(\forall i = \overline{1,I}; \forall j = \overline{1,I} :\right.$$
$$\left(\left({}^{I}\psi_i^j \cdot q_i \cdot q_j = 0\right) \wedge \left({}^{II}\psi_i^j \cdot q_i \cdot w_j = 0\right) \wedge\right.$$
$$\left.\left.\wedge \left({}^{III}\psi_i^j \cdot w_i \cdot w_j = 0\right)\right)\right)\}$$

to indicate a set of possible states of a process at stage n , where $y_n \in \mathbb{Y}_n$ is the state of a process at stage n, and $Y = \langle y_0, y_1, ..., y_N\rangle$ is the process trajectory. Before stage 1 there is one initial stage $y_p = y_0$. The set of possible decisions at stage n (when the process is in state y_{n-1}) can be described as follows:

$$\mathbb{S}_n(y_{n-1}) = \mathbb{S}_n\left(\langle n-1, q_1, ..., q_I, w_1, ..., w_I, D\rangle\right) =$$
$$= \{\langle s_1, ..., s_I\rangle \in \{0,1\}^I : \left(\forall i = \overline{1,I} : \left(s_i \le 1 - \mathrm{sgn}(q_i - 1)\right) \wedge\right.$$
$$\left.\wedge \left(s_i \le 1 - \mathrm{sgn}(w_i - 1)\right)\right) \wedge$$
$$\wedge \left(\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{d=n}^{\min\{T, n+G_i+r_i\}} \overline{O}_i \cdot s_i \cdot e_i^p(n,d) \le D\right) \wedge$$
$$\wedge \left(\forall i = \overline{1,I}; \forall j = \overline{1,I} : \left(\left(\left[q_j > 1\right] +\right.\right.\right.$$
$$\left. + s_i \cdot \left[G_i < q_j - 1\right] \le 2 - {}^{I}\psi_j^i\right) \wedge$$
$$\wedge \left(\mathrm{sgn}(w_j) + s_i \cdot \left(\left[G_i < w_j - 1 < G_i + r_i\right] +\right.\right.$$
$$\left.\left. + \left[w_j - 1 \le G_i < w_j - 1 + r_j\right]\right) \le 2 - {}^{I}\psi_j^i\right) \wedge$$
$$\wedge \left(s_j + s_i + \mathrm{sgn}\left(\left[G_j \le G_i < G_j + r_j\right] +\right.\right.$$
$$\left.\left. + \left[G_i < G_j < G_i + r_i\right]\right) \le 3 - {}^{I}\psi_j^i\right) \wedge$$
$$\wedge \left(\left[q_j > 0\right] + s_i \le 2 - {}^{II}\psi_j^i\right) \wedge$$
$$\wedge \left(\mathrm{sgn}(w_j) + s_i \cdot \left[w_j - 1 \ne G_i\right] \le 2 - {}^{II}\psi_j^i\right) \wedge$$
$$\wedge \left(\mathrm{s_i} + \mathrm{s_j} + \left[G_i \ne G_j\right] \le 3 - {}^{II}\psi_j^i\right) \wedge$$
$$\left.\left.\left.\wedge \left(\left[w_j > 1\right] + s_j + s_i \le 2 - {}^{III}\psi_j^i\right)\right)\right)\}$$

with $s_n \in \mathbb{S}_n(y_{n-1})$ decision taken at stage n, and $S = \langle s_1, \mathrm{s}_2, ..., \mathrm{s}_N\rangle$ being the control over the whole process.

We will use

$$g_n(y_{n-1}, \mathrm{s}_n) =$$
$$= g_n\left(\langle n-1, q_1, ..., q_I, w_1, ..., w_I, D\rangle, \langle s_1, ..., s_I\rangle\right) =$$
$$= \langle n, q_1^{'}, ..., q_I^{'}, \mathrm{w}_1^{'}, ..., \mathrm{w}_I^{'}, D^{'}\rangle)$$

to indicate a transformation function at stage n for decision $s_n \in \mathbb{S}_n(y_{n-1})$ at state $y_{n-1} \in \mathbb{Y}_{n-1}$ where:

$$q_i^{'} = \begin{cases} r_i \text{ for } w_i = 1 \\ q_i - 1 \text{ for } \mathrm{q}_i > 0 \\ 0 \text{ for } w_i = 0 \wedge \mathrm{q}_i = 0 \end{cases}$$

$$w_i^{'} = \begin{cases} G_i \text{ for } s_i = 1 \\ w_i - 1 \text{ for } \mathrm{w}_i > 0 \\ 0 \text{ for } s_i = 0 \wedge \mathrm{w}_i > 0 \end{cases} \qquad \left(i = \overline{1,I}\right)$$

$$D^{'} = D + \overline{\Gamma}(n) - \sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{d=1}^{\min\{T, \mathrm{n}+G_i+r_i\}} \overline{O}_i \cdot s_i \cdot e_i^p(\mathrm{n}, d)$$

We will use $f_n(y_{n-1}, \mathrm{s}_n)$ to indicate the cost of stage n undertaken when making a decision $s_n \in \mathbb{S}_n(y_{n-1})$ at state $y_{n-1} \in \mathbb{Y}_{n-1}$. The cost of stage n using the metric defined in (5) is as follows:

$$f_n(y_{n-1}, \mathrm{s}_n) =$$
$$= f_n\left(\langle n-1, q_1, ..., q_I, w_1, ..., w_I, D\rangle, \langle s_1, ..., s_I\rangle\right) =$$
$$= \sum_{p=1}^{P}\left(L_p(n) - \Phi_p(\mathcal{X}(n), \mathbb{S}) - \sum_{i=1}^{I}\left[q_i > 1\right] \cdot \overline{O}_i \cdot\right.$$
$$\cdot e_i^p\left(\sum_{z=1}^{n} z \cdot \left[G_i - n + z = q_i\right], \mathrm{n}\right) - \sum_{i=1}^{I}\left[w_i > 0\right] \cdot \overline{O}_i \cdot$$
$$\cdot e_i^p\left(\sum_{z=1}^{n} z \cdot \left[G_i - n + z + r_i = w_i\right], \mathrm{n}\right) -$$
$$\left. - \sum_{p=1}^{P}\sum_{i=1}^{I} \overline{O}_i \cdot s_i \cdot e_i^p(n,n)\right)$$

with $F(\mathbb{Y}, \mathbb{S})$ as the cost of decisions at all stages, using trajectory $\mathbb{Y}$ in case of making decisions S. $F(\mathbb{Y}, \mathbb{S})$ can be defined as:

$$F(\mathbb{Y}, \mathbb{S}) = \sum_{n-1}^{N} f_n(y_{n-1}, \mathrm{s}_n)$$

The algorithm for dynamic programming in the current optimization problem can be described as follows:

1. Let $n := T + 1$.
2. Add each state $y \in \mathbb{Y}_{n-1}$ features $s_n^*(y) = \langle 0, 0, ..., 0\rangle$ and $F_n(y) = 0$.
3. Let $n := n - 1$

4. Let each state $y \in \mathbb{Y}_{n-1}$ has decision $s_n^*(y) \in \mathbb{S}_n(y)$, for which
$$f_n\left(y, s_n^*(y)\right) + F_{n+1}\left(g_n\left(y, s_n^*(y)\right)\right) =$$
$$= \max_{s \in S_n(y)} \left[f_n(y, s) + F_{n+1}\left(g_n\left(y, s_n(y)\right)\right) \right]$$
$$\triangleq F_n(y)$$
5. Add each state $y \in \mathbb{Y}_{n-1}$ features $s_n^*(y)$ and $F_n(y)$.
6. If $n > 1$ then go to 2.
7. Beginning from state y_p construct a series of decisions $S^* = \left\langle s_1^*, s_2^*, ..., s_N^* \right\rangle$ and a trajectory $Y^* = \left\langle y_1^*, y_2^*, ..., y_N^* \right\rangle$, where
$$s_n^* = s_n^*\left(y_{n-1}^*\right),$$
$$y_n^* = g_n\left(y_{n-1}^*, s_n^*\left(y_{n-1}^*\right)\right),$$
$$\left(n = \overline{1, N}\right).$$
8. Let $F\left(Y^*, S^*\right) := F_1\left(y_p\right)$

In the formulated optimization problem, cost was the criterion function $\sum_{t=1}^{T} \gamma\left(t, \mathbb{S}\right)$. This method can be applied to any other metric as defined in the previous section $\sum_{n=1}^{T} \lambda(n, \mathbb{S})$.

4. Conclusions

This article presents a comprehensive information security model of a business organization. The model is time-based, because information security requires business continuity planning. The analysis shows that the problem of safeguard implementation can be resolved using a dynamic programming method. It is crucial for business organizations to plan and use budget by applying proper methods. Inefficient use of financial resources can cause an incorrect information security level. Our model and optimization method can be used to implement effective strategies to safeguard data. Dynamic programming was chosen because in comparison to the brute force search method it allows to solve bigger problems.

Table 1 shows the number of calculations required to obtain an optimal solution using the dynamic programming method and the brute force search method, depending on T with assumptions: $I = 2$, $\forall i = \overline{1, I} : r_i + G_i \geq T$.

Tab. 1. Comparison of dynamic programming and the brute force search method

T	Dynamic programming	Brute force search
1	4	4
2	32	16
3	108	64
4	256	256
5	500	1024
6	864	4096
7	1372	16384
8	2048	65536
9	2916	262144
10	4000	1048576
11	5324	4194304

Table 1 shows that the number of calculations using the brute force search method increases dramatically with the number of time periods. Dynamic programming, has better computational complexity and can be used to solve bigger problems.

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Deterministyczny model czasowy bezpieczeństwa informacyjnego organizacji biznesowej

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W artykule omówiono bezpieczeństwo informacyjne w organizacji biznesowej z wykorzystaniem deterministycznego modelu matematycznego opartego na przedziałach czasowych. Model odnosi się do kluczowych cech organizacji biznesowej z punktu widzenia bezpieczeństwa informacyjnego i oblicza poziom bezpieczeństwa informacyjnego w oparciu o miary ilościowe. Następnie wprowadzony model jest wykorzystany do oceny poziomu bezpieczeństwa informacyjnego, które może być osiągnięte dla znanych zagrożeń przy określonym budżecie. Z tego powodu został sformułowany problem optymalizacyjny wdrażania zabezpieczeń, a następnie przedstawiono sposób rozwiązania tego problemu oparty o metodę programowania dynamicznego. Dwie miary bezpieczeństwa informacyjnego, lokalna i globalna, zostały opisane w modelu matematycznym, natomiast jedna z miar została użyta w zadaniu optymalizacyjnym.

Słowa kluczowe: bezpieczeństwo informacyjne, model deterministyczny, programowanie dynamiczne.